Blitzer

Precalculus
5e

PRECALCULUS

5th EDITION

Robert Blitzer
Miami Dade College

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Library of Congress Cataloging-in-Publication Data

Blitzer, Robert.
Precalculus / Robert Blitzer Miami Dade College. —Fifth edition.
pages cm
Includes index.
ISBN 978-0-321-83734-9
1. Algebra—Textbooks. I. Title.

QA154.3.B586 2014
512—dc23

2012042264

1 2 3 4 5 6 7 8 9 10–DOW–16 15 14 13 12

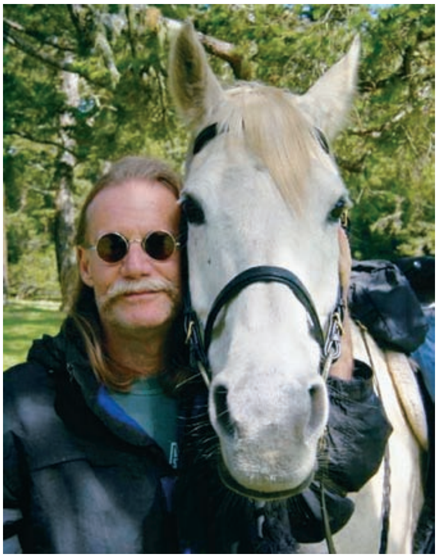
PEARSON
www.pearsonhighered.com

ISBN 10: 0-321-83734-7
ISBN 13: 978-0-321-83734-9

DEDICATION

For Jerid (1985–2012)

And for those who have loved their pets
and have been loved by them



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PREFACE

I've written ***Precalculus,* Fifth Edition**, to help diverse students, with different backgrounds and future goals, to succeed. The book has three fundamental goals:

1. To help students acquire a solid foundation in algebra and trigonometry, preparing them for other courses such as calculus, business calculus, and finite mathematics.
2. To show students how algebra and trigonometry can model and solve authentic real-world problems.
3. To enable students to develop problem-solving skills, while fostering critical thinking, within an interesting setting.

One major obstacle in the way of achieving these goals is the fact that very few students actually read their textbook. This has been a regular source of frustration for me and for my colleagues in the classroom. Anecdotal evidence gathered over years highlights two basic reasons that students do not take advantage of their textbook:

- "I'll never use this information."
- "I can't follow the explanations."

I've written every page of the Fifth Edition with the intent of eliminating these two objections. The ideas and tools I've used to do so are described for the student in "A Brief Guide to Getting the Most from This Book," which appears at the front of the book.

How Does *Precalculus* Differ from *Algebra and Trigonometry*?

Precalculus is not simply a condensed version of my Algebra and Trigonometry book. Precalculus students are different from algebra and trigonometry students, and this text reflects those differences. Here are a few examples:

- ***Algebra and Trigonometry*** devotes an entire chapter to linear equations, rational equations, quadratic equations, radical equations, linear inequalities, and developing models involving these equations and inequalities. ***Precalculus*** reviews these topics in three sections of the prerequisites chapter (P.7: Equations; P.8: Modeling with Equations; P.9: Linear Inequalities and Absolute Value Inequalities). Functions, the core of any precalculus course, are then introduced in Chapter 1.
- ***Precalculus*** contains a section on constructing functions from verbal descriptions and formulas (1.10: Modeling with Functions) that is not included in ***Algebra and Trigonometry***. Modeling skills are applied to situations that students are likely to see in calculus when solving applied problems involving maximum or minimum values.
- ***Precalculus*** develops trigonometry from the perspective of the unit circle (4.2: Trigonometric Functions: The Unit Circle). In ***Algebra and Trigonometry***, trigonometry is developed using right triangles.
- ***Precalculus*** contains a chapter (Chapter 11: Introduction to Calculus) that takes the student into calculus with discussions of limits, continuity, and derivatives. This chapter is not included in ***Algebra and Trigonometry***.
- Many of the liberal arts applications in ***Algebra and Trigonometry*** are replaced by more scientific or higher level applications in ***Precalculus***. Some examples:
    - Black Holes in Space (P.2: Exponents and Scientific Notation)
    - Average Velocity (1.5: More on Slope)
    - Newton's Law of Cooling (3.5: Exponential Growth and Decay; Modeling Data)
    - Modeling Involving Mixtures and Uniform Motion (7.1: Systems of Linear Equations in Two Variables)

What's New in the Fifth Edition?

New Applications and Real-World Data. I'm on a constant search for data that can be used to illustrate unique mathematical applications. I researched hundreds of books, magazines, newspapers, almanacs, and online sites to prepare the Fifth Edition. Among the 108 worked-out examples and exercises based on new data sets, you'll find applications involving modeling blood-alcohol concentration (Section P.1), starting salaries for college graduates (Section P.8), the world's vanishing tiger population (Section 2.3), and the year humans become immortal (Section 3.1).

Concept and Vocabulary Checks. The Fifth Edition contains 679 new short-answer exercises, mainly fill-in-the-blank and true/false items, that assess students' understanding of the definitions and concepts presented in each section. The Concept and Vocabulary Checks appear as separate features preceding the Exercise Sets.

Great Question! This feature takes the content of each Study Tip in the Fourth Edition and presents it in the context of a student question. Answers to questions offer suggestions for problem solving, point out common errors to avoid, and provide informal hints and suggestions. ‘Great Question!’ should draw students’ attention and curiosity more than the ‘Study Tips.’ As a secondary benefit, this new feature should help students not to feel anxious or threatened when asking questions in class.

New Chapter-Opening and Section-Opening Scenarios. Every chapter and every section open with a scenario based on an application, many of which are unique to the Fifth Edition. These scenarios are revisited in the course of the chapter or section in one of the book’s new examples, exercises, or discussions. The often humorous tone of these openers is intended to help fearful and reluctant students overcome their negative perceptions about math.

New Blitzer Bonuses. The Fifth Edition contains a variety of new but optional enrichment essays. Examples include “Using Algebra to Measure Blood-Alcohol Concentration” (Section P.1), “Seven Ways to Spend $1 Trillion” (Section P.2), “Addressing Stress Parabolically” (Section 2.2), “Five Things Scientists Learned from the Hubble Space Telescope” (Section 9.3), and “Ponzi Schemes and Geometric Sequences” (Section 10.3).

Sample Homework Assignments. Within each Exercise Set, I have chosen odd-numbered problems, primarily from the Practice Exercises, that can serve as sample homework assignments. These are indicated by a blue underline in the Annotated Instructor’s Edition. Based on the goals and objectives of your course, you may wish to enrich each sample homework assignment with additional exercises from the other categories in the Exercise Set.

New Interactive Figures. These new figures bring mathematical concepts to life and are included in MyMathLab. Used as a lecture tool, the figures help engage students more fully and save the time spent drawing figures by hand. Questions pertaining to each figure are assignable in MyMathLab and reinforce active learning and critical thinking. Each figure has an accompanying Exploratory Exercise that encourages further study and can be used as a presentation tool or as an open-ended learning assignment.

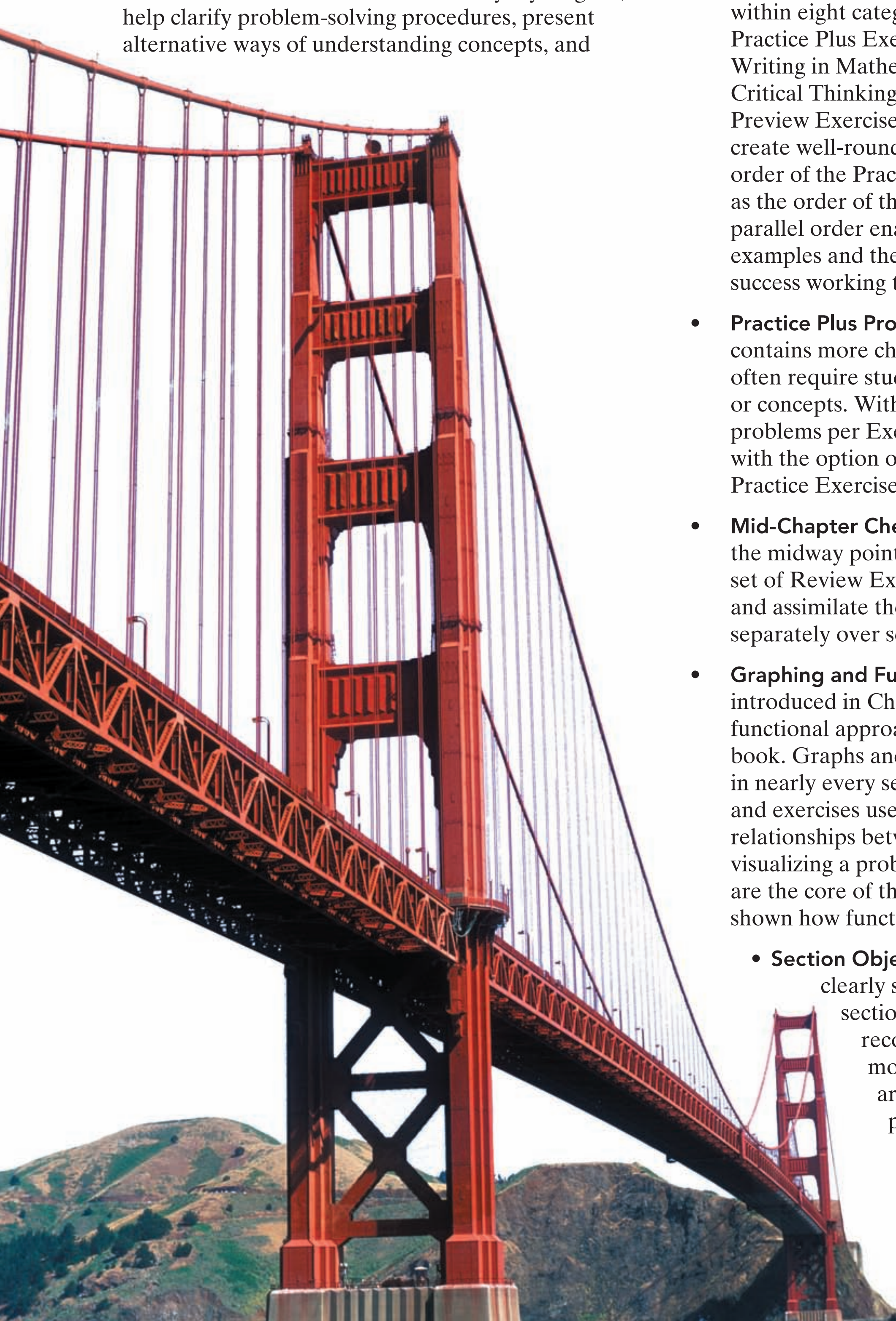
What Content and Organizational Changes Have Been Made to the Fifth Edition?

- **Section P.1 (Algebraic Expressions, Mathematical Models, and Real Numbers)** contains a new essay, now called a Blitzer Bonus, on using algebra to measure blood-alcohol concentration. This Blitzer Bonus should set the stage for the book’s engaging collection of unique applications.
- **Section P.6 (Rational Expressions)** presents a new example on excluding numbers from a rational expression with a trinomial denominator.

- **Section 1.6 (Transformations of Functions)** has a more thoroughly developed discussion of how stretching or shrinking changes a graph’s shape.
- **Section 1.7 (Combinations of Functions; Composite Functions)** has a new example on finding the domain of a function with a square root in the denominator. There is also a new example that ties in with the section opener (number of births and deaths in the United States) and illustrates an application of the algebra of functions.
- **Section 2.3 (Polynomial Functions and Their Graphs)** contains a new example on graphing $f(x) = -2(x - 1)^2(x + 2)$, a polynomial function whose equation is given in factored form.
- **Section 2.6 (Rational Functions and Their Graphs)** has a variety of exercises where students must factor to find vertical asymptotes or holes.
- **Section 2.7 (Polynomial and Rational Inequalities)** contains a new example on solving a polynomial inequality with irrational boundary points that requires the use of the quadratic formula.
- **Section 3.1 (Exponential Functions)** presents an intriguing new Blitzer Bonus on the year humans become immortal. The section also contains a new table clarifying interest plans in which interest is paid more than once a year.
- **Section 3.4 (Exponential and Logarithmic Equations)** has a new discussion (within the context of the Great Question! feature) on whether a negative number can belong to the solution set of a logarithmic equation.
- **Section 5.1 (Verifying Trigonometric Identities)** has a new discussion (within the context of the Great Question! feature) on the difference between solving a conditional equation and verifying that an equation is an identity.
- **Section 7.3 (Partial Fractions)** uses the Great Question! feature to include a discussion on speeding up the process of finding partial fraction decompositions.

What Familiar Features Have Been Retained in the Fifth Edition?

- **Detailed Worked-Out Examples.** Each worked example is titled, making clear the purpose of the example. Examples are clearly written and provide students with detailed step-by-step solutions. No steps are omitted and key steps are thoroughly explained to the right of the mathematics.
- **Explanatory Voice Balloons.** Voice balloons are used in a variety of ways to demystify mathematics. They translate mathematical ideas into everyday English, help clarify problem-solving procedures, present alternative ways of understanding concepts, and connect problem solving to concepts students have already learned.



- **Check Point Examples.** Each example is followed by a similar matched problem, called a Check Point, offering students the opportunity to test their understanding of the example by working a similar exercise. The answers to the Check Points are provided in the answer section.
- **Extensive and Varied Exercise Sets.** An abundant collection of exercises is included in an Exercise Set at the end of each section. Exercises are organized within eight category types: Practice Exercises, Practice Plus Exercises, Application Exercises, Writing in Mathematics, Technology Exercises, Critical Thinking Exercises, Group Exercises, and Preview Exercises. This format makes it easy to create well-rounded homework assignments. The order of the Practice Exercises is exactly the same as the order of the section's worked examples. This parallel order enables students to refer to the titled examples and their detailed explanations to achieve success working the Practice Exercises.
- **Practice Plus Problems.** This category of exercises contains more challenging practice problems that often require students to combine several skills or concepts. With an average of ten Practice Plus problems per Exercise Set, instructors are provided with the option of creating assignments that take Practice Exercises to a more challenging level.
- **Mid-Chapter Check Points.** At approximately the midway point in each chapter, an integrated set of Review Exercises allows students to review and assimilate the skills and concepts they learned separately over several sections.
- **Graphing and Functions.** Graphing and functions are introduced in Chapter 1, with an integrated graphing functional approach emphasized throughout the book. Graphs and functions that model data appear in nearly every section and Exercise Set. Examples and exercises use graphs of functions to explore relationships between data and to provide ways of visualizing a problem's solution. Because functions are the core of this course, students are repeatedly shown how functions relate to equations and graphs.
- **Section Objectives.** Learning objectives are clearly stated at the beginning of each section. These objectives help students recognize and focus on the section's most important ideas. The objectives are restated in the margin at their point of use.

- **Integration of Technology Using Graphic and Numerical Approaches to Problems.** Side-by-side features in the Technology boxes connect a problem's solution to graphic and numerical approaches to solving that problem. Although the use of graphing utilities is optional, students can use the explanatory voice balloons to understand different approaches to problems even if they are not using a graphing utility in the course.
- **Chapter Summaries.** Each chapter contains a review chart that summarizes the definitions and concepts in every section of the chapter. Examples that illustrate these key concepts are also referenced in the chart.
- **End-of-Chapter Materials.** A comprehensive collection of Review Exercises for each of the chapter's sections follows the Summary. This is followed by a Chapter Test that enables students to test their understanding of the material covered in the chapter. Beginning with Chapter 2, each chapter concludes with a comprehensive collection of mixed Cumulative Review Exercises.
- **Discovery.** Discovery boxes, found throughout the text, encourage students to further explore algebraic and trigonometric concepts. These explorations are optional and their omission does not interfere with the continuity of the topic at hand.

I hope that my passion for teaching, as well as my respect for the diversity of students I have taught and learned from over the years, is apparent throughout this new edition. By connecting algebra and trigonometry to the whole spectrum of learning, it is my intent to show students that their world is profoundly mathematical, and indeed, π is in the sky.

Robert Blitzer

Acknowledgments

An enormous benefit of authoring a successful series is the broad-based feedback I receive from the students, dedicated users, and reviewers. Every change to this edition is the result of their thoughtful comments and suggestions. I would like to express my appreciation to all the reviewers, whose collective insights form the backbone of this revision. In particular, I would like to thank the following people for reviewing *College Algebra*, *Algebra and Trigonometry*, *Precalculus*, and *Trigonometry*.

Barnhill, Kayoko Yates, Clark College
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Becan, Jaromir, University of Texas-San Antonio
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Van Veldhuizen, Philip, University of Nevada at Reno
Weaver, Jeffrey, Baton Rouge Community College
Wheeler, Amanda, Amarillo College
White, David, The Victoria College
Wienckowski, Tracy, University of Buffalo

Additional acknowledgments are extended to Dan Miller and Kelly Barber for preparing the solutions manuals; Brad Davis for preparing the answer section, serving as accuracy checker, and writing the new learning guide; cMPreparé formatting team for the book's brilliant paging; Brian Morris and Kevin Morris at Scientific Illustrators for superbly illustrating the book; Sheila Norman, photo researcher, for obtaining the book's new photographs; Rebecca Dunn, project manager, and Kathleen Manley, production editor, whose collective talents kept every aspect of this complex project moving through its many stages.

I would like to thank my editors at Pearson, Katie O'Connor and Dawn Murrin, who, with the assistance of Joseph Colella, guided and coordinated the book from manuscript through production. Thanks to Beth Paquin for the beautiful covers and interior design. Finally, thanks to Peggy Lucas for her innovative marketing efforts and to the entire Pearson sales force for their confidence and enthusiasm about the book.

Robert Blitzer

DYNAMIC RESOURCES

MyMathLab® Online Course (access code required)

MyMathLab delivers **proven results** in helping individual students succeed. It provides **engaging experiences** that personalize, stimulate, and measure learning for each student. And it comes from a **trusted partner** with educational expertise and an eye on the future.

To learn more about how MyMathLab combines proven learning applications with powerful assessment, visit **www.mymathlab.com** or contact your Pearson representative.

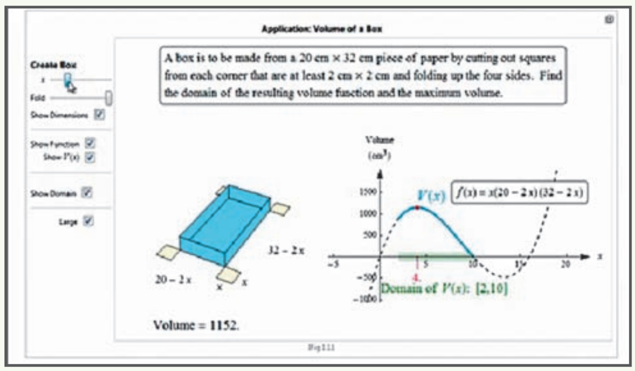
In Blitzer's **MyMathLab®** course, you have access to the most cutting-edge, innovative study solutions proven to increase student success. Noteworthy features include the following:

Ready to Go Courses.

These new courses provide students with all the same great MyMathLab features that you're used to but make it easier for instructors to get started. Each course includes author-chosen, preassigned homework, integrated review questions, quizzes, and cumulative review exercises to make creating your course even simpler.

Interactive Figures.

These *Mathematica*-based figures make the figures from the text come alive. Used during a lecture, interactive figures engage students more fully and save time that would otherwise be spent drawing them by hand. Exercises pertaining to each interactive figure are assignable in MyMathLab to reinforce active learning, critical thinking, and conceptual reasoning.



Integrated Review.

Skill review quizzes are assignable throughout the course, testing students on prerequisite knowledge. From these quizzes, each student receives a personalized, just-in-time review assignment, allowing them to refresh forgotten concepts.

MathTalk Videos.

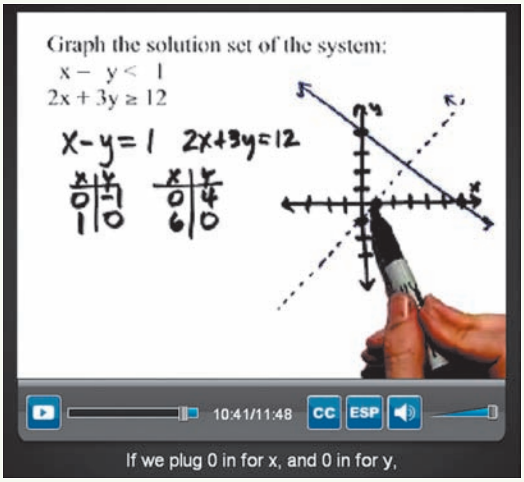
Engaging videos connect mathematics to real-life events and interesting applications. These fun, instructional videos show students that math is relevant to their daily lives and are assignable in MyMathLab.

Video Assessment.

Assignable MXL exercises are available for MathTalk videos to help students retain valuable information presented in the videos.

Section-Lecture Videos.

These videos provide lectures for each section of the text to help students review important concepts and procedures 24/7.



Concept and Vocabulary Check.

New and assignable in MyMathLab, these short-answer and fill-in-the blank exercises provide a quick check for understanding of concepts. These questions also test for reading comprehension before the student moves on to the exercises.

Chapter Test Prep Videos.

Students can watch instructors work through step-by-step solutions to all the Chapter Test exercises from the textbook. These are available in MyMathLab and on YouTube.

www.youtube.com/BlitzerPrecalculus5e

Instructor Resources

Additional resources can be downloaded from www.pearsonhighered.com or hardcopy resources can be ordered from your sales representative.

TestGen.

Enables instructors to build, edit, print, and administer tests using a computerized bank of algorithmic questions developed to cover all the objectives of the text.

PowerPoint Lecture Slides.

Fully editable lecture slides that correlate to the textbook.

Instructor's Solutions Manual.

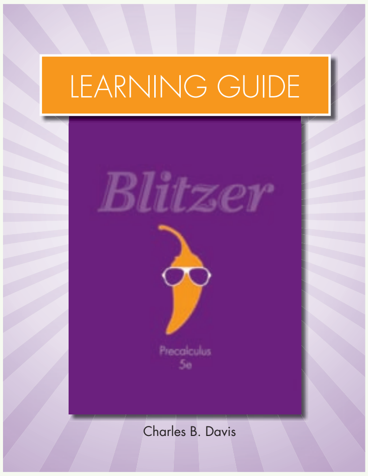
Fully worked solutions to all textbook exercises.

Mini Lecture Notes.

Additional examples and helpful teaching tips for each section.

Annotated Instructor's Edition.

Shorter answers are on the page beside the exercises. Longer answers are in the back of the text.



Student Resources

Additional resources to help student success are available to be packaged with the Blitzer textbook and MyMathLab access code.

Student's Solutions Manual.

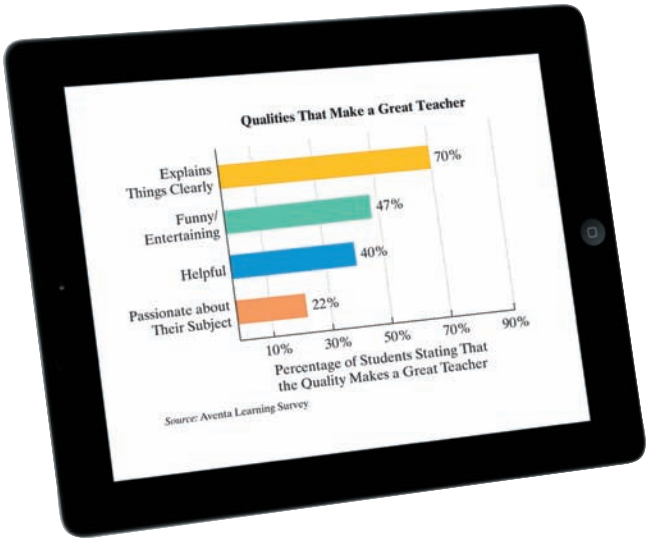
Fully worked solutions to odd-numbered exercises and available to be packaged with the textbook.

Learning Guide.

This note-taking guide is organized by objective and begins each chapter with an engaging application, providing additional examples and exercises for students to work through for greater conceptual understanding and mastery of mathematical topics. The Learning Guide is available as PDFs and customizable Word files in MyMathLab. They can also be packaged with the textbook and MyMathLab access code.

TO THE STUDENT

The bar graph shows some of the qualities that students say make a great teacher. It was my goal to incorporate each of these qualities throughout the pages of this book.



Explains Things Clearly

I understand that your primary purpose in reading *Precalculus* is to acquire a solid understanding of the required algebra and trigonometry topics in your precalculus course. In order to achieve this goal, I've carefully explained each topic. Important definitions and procedures are set off in boxes, and worked-out examples that present solutions in a step-by-step manner appear in every section. Each example is followed by a similar matched problem, called a Check Point, for you to try so that you can actively participate in the learning process as you read the book. (Answers to all Check Points appear in the back of the book.)

Funny & Entertaining

Who says that a precalculus textbook can't be entertaining? From our quirky cover to the photos in the chapter and section openers, prepare to expect the unexpected. I hope some of the book's enrichment essays, called Blitzer Bonuses, will put a smile on your face from time to time.

Helpful

I designed the book's features to help you acquire knowledge of algebra and trigonometry, as well as to show you how algebra and trigonometry can solve authentic problems that apply to your life. These helpful features include

- **Explanatory Voice Balloons:** Voice balloons are used in a variety of ways to make math less intimidating. They translate algebraic and trigonometric language into everyday English, help clarify problem-solving procedures, present alternative ways of understanding concepts, and connect new concepts to concepts you have already learned.
- **Great Question!:** The book's Great Question! boxes are based on questions students ask in class. The answers to these questions give suggestions for problem solving, point out common errors to avoid, and provide informal hints and suggestions.
- **Chapter Summaries:** Each chapter contains a review chart that summarizes the definitions and concepts in every section of the chapter. Examples from the chapter that illustrate these key concepts are also referenced in the chart. Review these summaries and you'll know the most important material in the chapter!

Passionate about the Subject

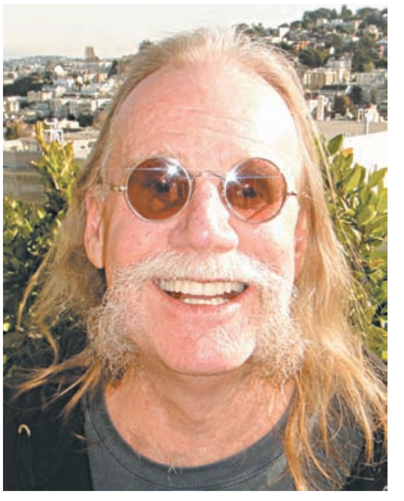
I passionately believe that no other discipline comes close to math in offering a more extensive set of tools for application and development of your mind. I wrote the book in Point Reyes National Seashore, 40 miles north of San Francisco. The park consists of 75,000 acres with miles of pristine surf-washed beaches, forested ridges, and bays bordered by white cliffs. It was my hope to convey the beauty and excitement of mathematics using nature's unspoiled beauty as a source of inspiration and creativity. Enjoy the pages that follow as you empower yourself with the algebra and trigonometry needed to succeed in college, your career, and your life.

Regards,

Bob

Robert Blitzer

ABOUT THE AUTHOR



Bob Blitzer is a native of Manhattan and received a Bachelor of Arts degree with dual majors in mathematics and psychology (minor: English literature) from the City College of New York. His unusual combination of academic interests led him toward a Master of Arts in mathematics from the University of Miami and a doctorate in behavioral sciences from Nova University. Bob's love for teaching mathematics was nourished for nearly 30 years at Miami Dade College, where he received numerous teaching awards, including Innovator of the Year from the League for Innovations in the Community College and an endowed chair based on excellence in the classroom. In addition to *Precalculus*, Bob has written textbooks covering introductory algebra, intermediate algebra, college algebra, algebra and trigonometry, trigonometry, and liberal arts mathematics, all published by Pearson. When not secluded in his Northern California writer's cabin, Bob can be found hiking the beaches and trails of Point Reyes National Seashore and tending to the chores required by his beloved entourage of horses, chickens, and irritable roosters.

APPLICATIONS INDEX

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PREREQUISITES: FUNDAMENTAL CONCEPTS OF ALGEBRA

CHAPTER P

What can algebra possibly have to tell me about

- the skyrocketing cost of a college education?
- my workouts?
- the effects of alcohol?
- the meaning of the national debt that exceeds $15 trillion?
- time dilation on a futuristic high-speed journey to a nearby star?
- ethnic diversity in the United States?
- the widening imbalance between numbers of women and men on college campuses?

This chapter reviews fundamental concepts of algebra that are prerequisites for the study of precalculus. Throughout the chapter, you will see how the special language of algebra describes your world.

HERE'S WHERE YOU'LL FIND THESE APPLICATIONS:

College costs: Section P.1, Example 2; Exercise Set P.1, Exercises 131–132

Workouts: Exercise Set P.1, Exercises 129–130

The effects of alcohol: Blitzer Bonus on page 15

The national debt: Section P.2, Example 6

Time dilation: Blitzer Bonus on page 44

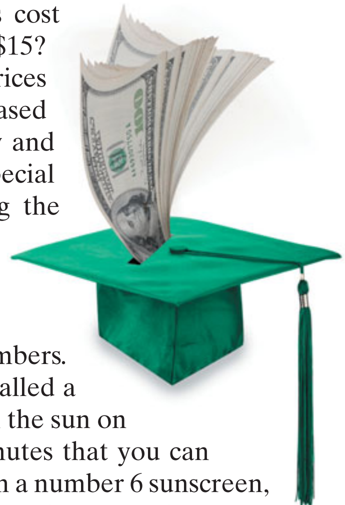
U.S. ethnic diversity: Chapter P Review, Exercise 23

College gender imbalance: Chapter P Test, Exercise 32.

SECTION P.1 Algebraic Expressions, Mathematical Models, and Real Numbers

Objectives

1. Evaluate algebraic expressions.
2. Use mathematical models.
3. Find the intersection of two sets.
4. Find the union of two sets.
5. Recognize subsets of the real numbers.
6. Use inequality symbols.
7. Evaluate absolute value.
8. Use absolute value to express distance.
9. Identify properties of the real numbers.
10. Simplify algebraic expressions.



How would your lifestyle change if a gallon of gas cost \$9.15? Or if the price of a staple such as milk was \$15? That's how much those products would cost if their prices had increased at the same rate college tuition has increased since 1980. (*Source:* Center for College Affordability and Productivity) In this section, you will learn how the special language of algebra describes your world, including the skyrocketing cost of a college education.

Algebraic Expressions

Algebra uses letters, such as x and y, to represent numbers. If a letter is used to represent various numbers, it is called a **variable**. For example, imagine that you are basking in the sun on the beach. We can let x represent the number of minutes that you can stay in the sun without burning with no sunscreen. With a number 6 sunscreen, exposure time without burning is six times as long, or 6 times x. This can be written $6 \cdot x$, but it is usually expressed as $6x$. Placing a number and a letter next to one another indicates multiplication.

Notice that $6x$ combines the number 6 and the variable x using the operation of multiplication. A combination of variables and numbers using the operations of addition, subtraction, multiplication, or division, as well as powers or roots, is called an **algebraic expression**. Here are some examples of algebraic expressions:

$$x + 6, \quad x - 6, \quad 6x, \quad \frac{x}{6}, \quad 3x + 5, \quad x^2 - 3, \quad \sqrt{x} + 7.$$

Many algebraic expressions involve *exponents*. For example, the algebraic expression

$$4x^2 + 341x + 3194$$

approximates the average cost of tuition and fees at public U.S. colleges for the school year ending x years after 2000. The expression x^2 means $x \cdot x$ and is read "x to the second power" or "x squared." The exponent, 2, indicates that the base, x, appears as a factor two times.

Exponential Notation

If n is a counting number (1, 2, 3, and so on),

Exponent or Power

$$b^n = \underbrace{b \cdot b \cdot b \cdot \cdots \cdot b}_{b \text{ appears as a factor } n \text{ times.}}$$

Base

b^n is read "the nth power of b" or "b to the nth power." Thus, the nth power of b is defined as the product of n factors of b. The expression b^n is called an **exponential expression**. Furthermore, $b^1 = b$.

For example,

$$8^2 = 8 \cdot 8 = 64, \quad 5^3 = 5 \cdot 5 \cdot 5 = 125, \quad \text{and} \quad 2^4 = 2 \cdot 2 \cdot 2 \cdot 2 = 16.$$

1 Evaluate algebraic expressions.

Evaluating Algebraic Expressions

Evaluating an algebraic expression means to find the value of the expression for a given value of the variable.

Many algebraic expressions involve more than one operation. Evaluating an algebraic expression without a calculator involves carefully applying the following order of operations agreement:

The Order of Operations Agreement

1. Perform operations within the innermost parentheses and work outward. If the algebraic expression involves a fraction, treat the numerator and the denominator as if they were each enclosed in parentheses.
2. Evaluate all exponential expressions.
3. Perform multiplications and divisions **as they occur**, working **from left to right**.
4. Perform additions and subtractions **as they occur**, working **from left to right**.

EXAMPLE 1 Evaluating an Algebraic Expression

Evaluate $7 + 5(x - 4)^3$ for $x = 6$.

SOLUTION

$$7 + 5(x - 4)^3 = 7 + 5(6 - 4)^3 \quad \text{Replace } x \text{ with 6.}$$
$$= 7 + 5(2)^3 \quad \text{First work inside parentheses: } 6 - 4 = 2.$$
$$= 7 + 5(8) \quad \text{Evaluate the exponential expression: } 2^3 = 2 \cdot 2 \cdot 2 = 8.$$
$$= 7 + 40 \quad \text{Multiply: } 5(8) = 40.$$
$$= 47 \quad \text{Add.}$$

Check Point 1 Evaluate $8 + 6(x - 3)^2$ for $x = 13$.

2 Use mathematical models.

Formulas and Mathematical Models

An **equation** is formed when an equal sign is placed between two algebraic expressions. One aim of algebra is to provide a compact, symbolic description of the world. These descriptions involve the use of *formulas*. A **formula** is an equation that uses variables to express a relationship between two or more quantities.

Here are two examples of formulas related to heart rate and exercise.

Couch-Potato Exercise

$$H = \frac{1}{5}(220 - a)$$

Heart rate, in beats per minute, | is | $\frac{1}{5}$ of | the difference between 220 and your age.

Working It

$$H = \frac{9}{10}(220 - a)$$

Heart rate, in beats per minute, | is | $\frac{9}{10}$ of | the difference between 220 and your age.

The process of finding formulas to describe real-world phenomena is called **mathematical modeling**. Such formulas, together with the meaning assigned to the variables, are called **mathematical models**. We often say that these formulas model, or describe, the relationships among the variables.

EXAMPLE 2 Modeling the Cost of Attending a Public College

The bar graph in **Figure P.1** shows the average cost of tuition and fees for public four-year colleges, adjusted for inflation. The formula

$$T = 4x^2 + 341x + 3194$$

models the average cost of tuition and fees, T, for public U.S. colleges for the school year ending x years after 2000.

a. Use the formula to find the average cost of tuition and fees at public U.S. colleges for the school year ending in 2010.

b. By how much does the formula underestimate or overestimate the actual cost shown in **Figure P.1**?

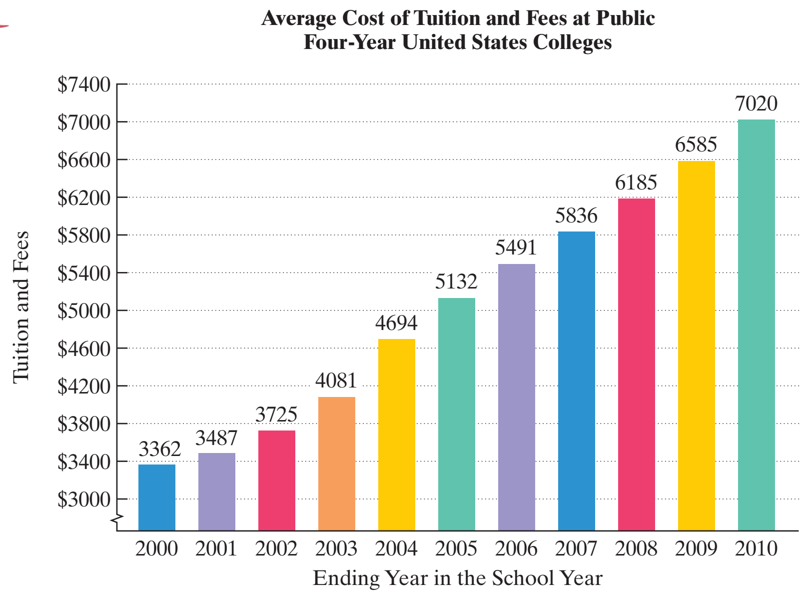


FIGURE P.1
Source: The College Board

SOLUTION

a. Because 2010 is 10 years after 2000, we substitute 10 for x in the given formula. Then we use the order of operations to find T, the average cost of tuition and fees for the school year ending in 2010.

$T = 4x^2 + 341x + 3194$	This is the given mathematical model.
$T = 4(10)^2 + 341(10) + 3194$	Replace each occurrence of x with 10.
$T = 4(100) + 341(10) + 3194$	Evaluate the exponential expression: $10^2 = 10 \cdot 10 = 100$.
$T = 400 + 3410 + 3194$	Multiply from left to right: $4(100) = 400$ and $341(10) = 3410$.
$T = 7004$	Add.

The formula indicates that for the school year ending in 2010, the average cost of tuition and fees at public U.S. colleges was \$7004.

b. **Figure P.1** shows that the average cost of tuition and fees for the school year ending in 2010 was \$7020.

The cost obtained from the formula, \$7004, underestimates the actual data value by \$7020 − \$7004, or by \$16.

Check Point 2 Assuming trends indicated by the data in **Figure P.1** continue, use the formula $T = 4x^2 + 341x + 3194$, described in Example 2, to project the average cost of tuition and fees at public U.S. colleges for the school year ending in 2015.

Sometimes a mathematical model gives an estimate that is not a good approximation or is extended to include values of the variable that do not make sense. In these cases, we say that **model breakdown** has occurred. For example, it is not likely that the formula in Example 2 would give a good estimate of tuition and fees in 2050 because it is too far in the future. Thus, model breakdown would occur.

Sets

Before we describe the set of real numbers, let's be sure you are familiar with some basic ideas about sets. A **set** is a collection of objects whose contents can be clearly determined. The objects in a set are called the **elements** of the set. For example, the set of numbers used for counting can be represented by

$$\{1, 2, 3, 4, 5, \ldots\}.$$

The braces, { }, indicate that we are representing a set. This form of representation, called the **roster method**, uses commas to separate the elements of the set. The symbol consisting of three dots after the 5, called an *ellipsis*, indicates that there is no final element and that the listing goes on forever.

A set can also be written in **set-builder notation**. In this notation, the elements of the set are described but not listed. Here is an example:

$$\{x \mid x \text{ is a counting number less than } 6\}.$$

The set of all x | such that | x is a counting number less than 6.

The same set written using the roster method is

$$\{1, 2, 3, 4, 5\}.$$

GREAT QUESTION!

Can I use symbols other than braces when writing sets using the roster method?

No. Grouping symbols such as parentheses, ( ), and square brackets, [ ], are not used to represent sets in the roster method. Furthermore, only commas are used to separate the elements of a set. Separators such as colons or semicolons are not used.

3 Find the intersection of two sets.

If A and B are sets, we can form a new set consisting of all elements that are in both A and B. This set is called the *intersection* of the two sets.

Definition of the Intersection of Sets

The **intersection** of sets A and B, written $A \cap B$, is the set of elements common to both set A **and** set B. This definition can be expressed in set-builder notation as follows:

$$A \cap B = \{x \mid x \text{ is an element of } A \text{ AND } x \text{ is an element of } B\}.$$

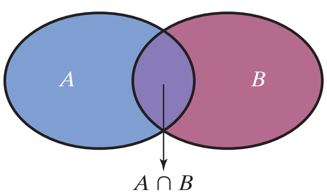


FIGURE P.2 Picturing the intersection of two sets

Figure P.2 shows a useful way of picturing the intersection of sets A and B. The figure indicates that $A \cap B$ contains those elements that belong to both A and B at the same time.

EXAMPLE 3 Finding the Intersection of Two Sets

Find the intersection: $\{7, 8, 9, 10, 11\} \cap \{6, 8, 10, 12\}$.

SOLUTION

The elements common to $\{7, 8, 9, 10, 11\}$ and $\{6, 8, 10, 12\}$ are 8 and 10. Thus,

$$\{7, 8, 9, 10, 11\} \cap \{6, 8, 10, 12\} = \{8, 10\}.$$

Check Point 3 Find the intersection: $\{3, 4, 5, 6, 7\} \cap \{3, 7, 8, 9\}$.

If a set has no elements, it is called the **empty set**, or the **null set**, and is represented by the symbol $\varnothing$ (the Greek letter phi). Here is an example that shows how the empty set can result when finding the intersection of two sets:

$$\{2, 4, 6\} \cap \{3, 5, 7\} = \varnothing.$$

These sets have no common elements.

Their intersection has no elements and is the empty set.

4 Find the union of two sets.

Another set that we can form from sets A and B consists of elements that are in A or B or in both sets. This set is called the *union* of the two sets.

Definition of the Union of Sets

The **union** of sets A and B, written $A \cup B$, is the set of elements that are members of set A **or** of set B or of both sets. This definition can be expressed in set-builder notation as follows:

$$A \cup B = \{x \mid x \text{ is an element of } A \text{ OR } x \text{ is an element of } B\}.$$

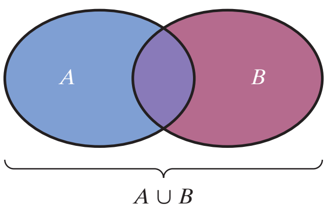


FIGURE P.3 Picturing the union of two sets

Figure P.3 shows a useful way of picturing the union of sets A and B. The figure indicates that $A \cup B$ is formed by joining the sets together.

We can find the union of set A and set B by listing the elements of set A. Then we include any elements of set B that have not already been listed. Enclose all elements that are listed with braces. This shows that the union of two sets is also a set.

GREAT QUESTION!

How can I use the words *union* and *intersection* to help me distinguish between these two operations?

Union, as in a marriage union, suggests joining things, or uniting them. Intersection, as in the intersection of two crossing streets, brings to mind the area common to both, suggesting things that overlap.

EXAMPLE 4 Finding the Union of Two Sets

Find the union: $\{7, 8, 9, 10, 11\} \cup \{6, 8, 10, 12\}$.

SOLUTION

To find $\{7, 8, 9, 10, 11\} \cup \{6, 8, 10, 12\}$, start by listing all the elements from the first set, namely, 7, 8, 9, 10, and 11. Now list all the elements from the second set that are not in the first set, namely, 6 and 12. The union is the set consisting of all these elements. Thus,

$$\{7, 8, 9, 10, 11\} \cup \{6, 8, 10, 12\} = \{6, 7, 8, 9, 10, 11, 12\}.$$

Although 8 and 10 appear in both sets, do not list 8 and 10 twice.

Check Point 4 Find the union: $\{3, 4, 5, 6, 7\} \cup \{3, 7, 8, 9\}$.

5 Recognize subsets of the real numbers.

The Set of Real Numbers

The sets that make up the real numbers are summarized in **Table P.1** at the top of the next page. We refer to these sets as **subsets** of the real numbers, meaning that all elements in each subset are also elements in the set of real numbers.

Notice the use of the symbol $\approx$ in the examples of irrational numbers. The symbol means "is approximately equal to." Thus,

$$\sqrt{2} \approx 1.414214.$$

We can verify that this is only an approximation by multiplying 1.414214 by itself. The product is very close to, but not exactly, 2:

$$1.414214 \times 1.414214 = 2.000001237796.$$

TECHNOLOGY

A calculator with a square root key gives a decimal approximation for $\sqrt{2}$, not the exact value.

Table P.1 Important Subsets of the Real Numbers

Name/Symbol	Description	Examples
Natural numbers $\mathbb{N}$	$\{1, 2, 3, 4, 5, \ldots\}$ These are the numbers that we use for counting.	$2, 3, 5, 17$
Whole numbers $\mathbb{W}$	$\{0, 1, 2, 3, 4, 5, \ldots\}$ The set of whole numbers includes 0 and the natural numbers.	$0, 2, 3, 5, 17$
Integers $\mathbb{Z}$	$\{\ldots, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, \ldots\}$ The set of integers includes the negatives of the natural numbers and the whole numbers.	$-17, -5, -3, -2, 0, 2, 3, 5, 17$
Rational numbers $\mathbb{Q}$	$\left\{\frac{a}{b} \,\middle\vert\, a \text{ and } b \text{ are integers and } b \neq 0\right\}$ This means that b is not equal to zero. The set of rational numbers is the set of all numbers that can be expressed as a quotient of two integers, with the denominator not 0. Rational numbers can be expressed as terminating or repeating decimals.	$-17 = \frac{-17}{1}, -5 = \frac{-5}{1}, -3, -2,$ $0, 2, 3, 5, 17,$ $\frac{2}{5} = 0.4,$ $\frac{-2}{3} = -0.6666\ldots = -0.\overline{6}$
Irrational numbers $\mathbb{I}$	The set of irrational numbers is the set of all numbers whose decimal representations are neither terminating nor repeating. Irrational numbers cannot be expressed as a quotient of integers.	$\sqrt{2} \approx 1.414214$ $-\sqrt{3} \approx -1.73205$ $\pi \approx 3.142$ $-\frac{\pi}{2} \approx -1.571$

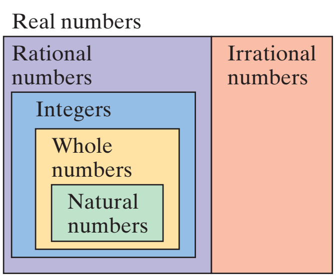


FIGURE P.4 Every real number is either rational or irrational.

Not all square roots are irrational. For example, $\sqrt{25} = 5$ because $5^2 = 5 \cdot 5 = 25$. Thus, $\sqrt{25}$ is a natural number, a whole number, an integer, and a rational number $\left(\sqrt{25} = \frac{5}{1}\right)$.

The set of *real numbers* is formed by taking the union of the sets of rational numbers and irrational numbers. Thus, every real number is either rational or irrational, as shown in **Figure P.4**.

Real Numbers

The set of **real numbers** is the set of numbers that are either rational or irrational:

$$\{x \mid x \text{ is rational or } x \text{ is irrational}\}.$$

The symbol $\mathbb{R}$ is used to represent the set of real numbers. Thus,

$$\mathbb{R} = \{x \mid x \text{ is rational}\} \cup \{x \mid x \text{ is irrational}\}.$$

EXAMPLE 5 Recognizing Subsets of the Real Numbers

Consider the following set of numbers:

$$\left\{-7, -\frac{3}{4}, 0, 0.\overline{6}, \sqrt{5}, \pi, 7.3, \sqrt{81}\right\}.$$

List the numbers in the set that are

a. natural numbers. **b.** whole numbers. **c.** integers.
d. rational numbers. **e.** irrational numbers. **f.** real numbers.

SOLUTION

a. Natural numbers: The natural numbers are the numbers used for counting. The only natural number in the set $\left\{-7, -\frac{3}{4}, 0, 0.\overline{6}, \sqrt{5}, \pi, 7.3, \sqrt{81}\right\}$ is $\sqrt{81}$ because $\sqrt{81} = 9$. (9 multiplied by itself, or 9^2, is 81.)

b. Whole numbers: The whole numbers consist of the natural numbers and 0. The elements of the set $\{-7, -\frac{3}{4}, 0, 0.\overline{6}, \sqrt{5}, \pi, 7.3, \sqrt{81}\}$ that are whole numbers are 0 and $\sqrt{81}$.

c. Integers: The integers consist of the natural numbers, 0, and the negatives of the natural numbers. The elements of the set $\{-7, -\frac{3}{4}, 0, 0.\overline{6}, \sqrt{5}, \pi, 7.3, \sqrt{81}\}$ that are integers are $\sqrt{81}$, 0, and -7.

d. Rational numbers: All numbers in the set $\{-7, -\frac{3}{4}, 0, 0.\overline{6}, \sqrt{5}, \pi, 7.3, \sqrt{81}\}$ that can be expressed as the quotient of integers are rational numbers. These include $-7\left(-7 = \frac{-7}{1}\right), -\frac{3}{4}, 0\left(0 = \frac{0}{1}\right)$, and $\sqrt{81}\left(\sqrt{81} = \frac{9}{1}\right)$. Furthermore, all numbers in the set that are terminating or repeating decimals are also rational numbers. These include $0.\overline{6}$ and 7.3.

e. Irrational numbers: The irrational numbers in the set $\{-7, -\frac{3}{4}, 0, 0.\overline{6}, \sqrt{5}, \pi, 7.3, \sqrt{81}\}$ are $\sqrt{5}\left(\sqrt{5} \approx 2.236\right)$ and $\pi(\pi \approx 3.14)$. Both $\sqrt{5}$ and π are only approximately equal to 2.236 and 3.14, respectively. In decimal form, $\sqrt{5}$ and π neither terminate nor have blocks of repeating digits.

f. Real numbers: All the numbers in the given set $\{-7, -\frac{3}{4}, 0, 0.\overline{6}, \sqrt{5}, \pi, 7.3, \sqrt{81}\}$ are real numbers. • • •

Check Point 5 Consider the following set of numbers:

$$\left\{-9, -1.3, 0, 0.\overline{3}, \frac{\pi}{2}, \sqrt{9}, \sqrt{10}\right\}.$$

List the numbers in the set that are

a. natural numbers. **b.** whole numbers. **c.** integers.
d. rational numbers. **e.** irrational numbers. **f.** real numbers.

The Real Number Line

The **real number line** is a graph used to represent the set of real numbers. An arbitrary point, called the **origin**, is labeled 0. Select a point to the right of 0 and label it 1. The distance from 0 to 1 is called the **unit distance**. Numbers to the right of the origin are **positive** and numbers to the left of the origin are **negative**. The real number line is shown in **Figure P.5**.

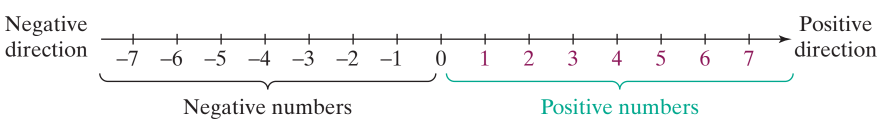


FIGURE P.5 The real number line

Real numbers are **graphed** on a number line by placing a dot at the correct location for each number. The integers are easiest to locate. In **Figure P.6**, we've graphed six rational numbers and three irrational numbers on a real number line.

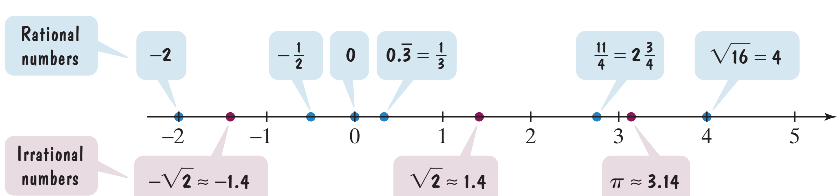


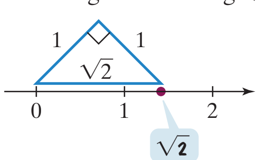
FIGURE P.6 Graphing numbers on a real number line

Every real number corresponds to a point on the number line and every point on the number line corresponds to a real number. We say that there is a **one-to-one correspondence** between all the real numbers and all points on a real number line.

GREAT QUESTION!

How did you locate $\sqrt{2}$ as a precise point on the number line in Figure P.6?

We used a right triangle with two legs of length 1. The remaining side has a length measuring $\sqrt{2}$.



We'll have lots more to say about right triangles later in the book.

6 Use inequality symbols.

Ordering the Real Numbers

On the real number line, the real numbers increase from left to right. The lesser of two real numbers is the one farther to the left on a number line. The greater of two real numbers is the one farther to the right on a number line.

Look at the number line in **Figure P.7**. The integers -4 and -1 are graphed.

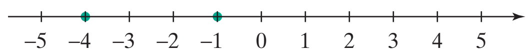


FIGURE P.7

Observe that -4 is to the left of -1 on the number line. This means that -4 is less than -1.

$$-4 < -1$$

-4 is less than -1 because -4 is to the **left** of -1 on the number line.

In **Figure P.7**, we can also observe that -1 is to the right of -4 on the number line. This means that -1 is greater than -4.

$$-1 > -4$$

-1 is greater than -4 because -1 is to the **right** of -4 on the number line.

The symbols $<$ and $>$ are called **inequality symbols**. These symbols always point to the lesser of the two real numbers when the inequality statement is true.

-4 is less than -1. $\quad -4 < -1 \quad$ The symbol points to -4, the lesser number.

-1 is greater than -4. $\quad -1 > -4 \quad$ The symbol still points to -4, the lesser number.

The symbols $<$ and $>$ may be combined with an equal sign, as shown in the following table:

This inequality is true if either the $<$ part or the $=$ part is true.

This inequality is true if either the $>$ part or the $=$ part is true.

Symbols	Meaning	Examples	Explanation
$a \le b$	a is less than or equal to b.	$2 \le 9$ $9 \le 9$	Because $2 < 9$ Because $9 = 9$
$b \ge a$	b is greater than or equal to a.	$9 \ge 2$ $2 \ge 2$	Because $9 > 2$ Because $2 = 2$

7 Evaluate absolute value.

Absolute Value

The **absolute value** of a real number a, denoted by $|a|$, is the distance from 0 to a on the number line. This distance is always taken to be nonnegative. For example, the real number line in **Figure P.8** shows that

$$|-3| = 3 \quad \text{and} \quad |5| = 5.$$

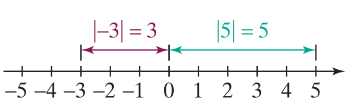


FIGURE P.8 Absolute value as the distance from 0

The absolute value of -3 is 3 because -3 is 3 units from 0 on the number line. The absolute value of 5 is 5 because 5 is 5 units from 0 on the number line. The absolute value of a positive real number or 0 is the number itself. The absolute value of a negative real number, such as -3, is the number without the negative sign.